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TEACHER LEARNING IN MATHEMATICS: USING STUDENT WORK TO PROMOTE COLLECTIVE INQUIRY

ABSTRACT. The study describes teachers' collective work in which they developed deeper understanding of their own students' mathematical thinking. Teachers at one school met in monthly workgroups throughout the year. Prior to each workgroup, they posed a similar mathematical problem to their students. The workgroup discussions centered on the student work those problems generated. This study draws on a *transformation of participation* perspective to address the questions: What do teachers learn through collective examination of student work? How is teacher learning evident in shifts in participation in discussions centered on student work? The analyses account for the learning of the group by documenting key shifts in teachers' participation across the year. The first shift in participation occurred when teachers as a group learned to attend to the details of children's thinking. A second shift in participation occurred as teachers began to develop possible instructional trajectories in mathematics. We focus our discussion on the significance of the use of student work and a transformation of participation view in analyzing the learning trajectory of teachers as a group.

KEY WORDS: children's mathematical thinking, professional development, school-wide inquiry, sociocultural theory, student work, teacher learning

A large body of literature has demonstrated that supporting teachers to meet the ambitious and complex visions of mathematics reform is difficult (e.g., Borko et al., 1992; Jaworski, 1994; Kazemi & Stipek, 2001; Schifter, 1998). Because of the inherent complexity of understanding how and what teachers learn, Wilson and Berne (1999) have called on researchers to study professional development rooted in teachers' own practice. Organizing teacher learning around the study of student work is one particular way in which professional development can be situated in practice (Ball & Cohen, 1999; Lin, 2002; Little, 1999).

In this article, we describe an approach to professional development in which teachers used their students' mathematical work as a focus for their collective inquiry (Franke et al., 1998; Richardson, 1990). The purpose of our analysis is to account for the learning of teachers as a group. To do this, we draw on sociocultural theories of learning that define learning as the *transformation of participation* (Rogoff, 1997). In the research reported, we address the following questions: What do teachers learn

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through collective examination of student work? How is teacher learning evident in shifts in participation in discussions centered on student work? Our findings lead us to conjecture about the use of student work as a mediator of teacher learning.

BACKGROUND

A number of recent publications have advocated using student work as a tool for professional development (e.g., Ball & Cohen, 1999; Little, 2002). This use of student work has the potential to influence professional discourse about teaching and learning, to engage teachers in a cycle of experimentation and reflection and to shift teachers' focus from one of general pedagogy to one that is particularly connected to their own students. Whether these opportunities are realized depends on the actual use of student work in professional activity. Empirical research on the use of student work is limited, given its relatively new emergence as a mechanism for promoting professional development. It is a component of Japanese Lesson Study[1] and alternative versions (Fernandez, Cannon, & Chokshi, 2003) and is featured in protocols developed by the Coalition of Essential Schools and Harvard Project Zero (Blythe, Allen, & Powell, 1999). Examining student work is also a component of several new case-based approaches to professional development in mathematics such as Developing Mathematical Ideas (Schifter, Bastable, & Russell, 1999), the Algebraic Thinking Toolkit (Driscoll, 1999) and the QUASAR cases on cognitive demand (Stein, Smith, Henningsen & Silver, 2000).

In a recent review of 26 published reports and papers in the United States, Little (in press) found only a handful of studies that constructed detailed observational records of teacher interactions around student work. Those studies suggest that simply bringing together teachers to "look at student work" did not necessarily open up opportunities for learning. How student work was used, the ways classrooms were represented in teacher talk, and the norms and habits of professional discourse influenced the potential impact on teacher learning and knowledge (see also Crockett, 2002; Crespo, 2002). As the use of student work in various professional development communities grows, the need to examine how student work is used to focus teacher inquiry heightens. We contribute to this need by offering a way of documenting teacher learning that examining student work supports.

CONCEPTUAL FRAMEWORK

Our analyses are guided by a situated view of learning. Understanding learning, as it emerges in activity, is paramount to such a perspective (Greeno & Middle School Mathematics Through Applications Project, 1998). This perspective centers on how people engage in routine activity and the role things such as tools and participation structures play in the practices that evolve (Wertsch, 1998). We apply a *transformation of participation* view, as described by Rogoff (1997), Lave (1996) and Wenger (1998), to account for a group's collective examination of student work.

Rogoff (1997) explains a transformation of participation view of learning by contrasting it with two other models of learning: acquisition and transmission. Both models assume a boundary between the world and individual; the former posits that individuals receive information transmitted from their environment while the latter posits that the environment inserts information into the individual. The transformation of participation view takes neither the environment nor the individual as the unit of analysis. Instead, it holds activity as the primary unit of analysis and accounts for individual development by examining how individuals engage in interpersonal and cultural-historical activities. Rogoff (1997) provides the following explication:

> . . . a person develops through participation in an activity, changing to be involved in the situation at hand in ways that contribute both to the ongoing event and to the person's preparation for involvement in other similar events. Instead of studying a person's possession or acquisition of a capacity or a bit of knowledge, the focus is on people's active changes in understanding, facility, and motivation involved in an unfolding event or activity in which they participate (p. 271).

The shifts in participation do not merely mark changes in activity or behavior. Shifts in participation involve a transformation of roles and the crafting of new identities, identities that are linked to new knowledge and skill (Wenger, 1998). Lave (1996) states, ". . . crafting identities is a *social* process, and becoming more knowledgeably skilled is an aspect of participation in social practice . . . who you are becoming shapes crucially and fundamentally what you 'know' " (p. 157). Our use of this conceptual framework in this article is an example of what Rogoff (1997) terms the interpersonal level of analysis in that it "focuses on how people communicate and coordinate efforts in face-to-face interaction . . ." (p. 269). This focus on the interpersonal level leads us to give primacy to the way the practice of the teacher group evolved, while we keep individual contributions and the larger social context of professional development practices in the background. Analyzing teachers' collective engagement with student work, then, reveals not only their deeper knowledge about student thinking and mathematics but also their developing professional identities as teachers. This attention to participation and identity has further implications for the kinds of practices that teachers pursue with one another and with their students.

METHODOLOGY

This study uses data from a workgroup of ten teachers who met regularly across the academic year in order to document the way the group's practice developed during that first year. It is beyond the scope of the paper to provide a full account of why it has been sustained to the present (see Franke & Kazemi, 2001). Our main contribution in this article is to provide an analytic frame for understanding teacher learning as shifts in participation. We engaged teachers from one elementary school in ongoing professional development that consisted of two main components: (a) facilitated workgroup meetings centered on students' written or oral mathematical work; and (b) observations and informal interactions with teachers in their classrooms.

The design of the professional development was modified based on pilot work using a Cognitively Guided Instruction [CGI] (Carpenter, Fennema, Franke, Levi, & Empson, 1999) model of professional development at two other elementary schools in the 1996–1997 school year. We did not follow the CGI approach by conducting workshops with the teachers and presenting them with the frameworks, nor did we design activities using videos or worksheets for them to make sense of how the typologies for problem types and strategies related to one another. Instead, we introduced CGI principles and terminology as teachers made observations of their own students' mathematical thinking. However, we did provide teachers with common problems to use in their classes that consisted of CGI word problem types (e.g., join change unknown[2] or a missing addend problem, see Table III for examples).

Setting and Participants

The study took place at Crestview Elementary School (all names are pseudonyms) in a small urban school district. Data were collected during the 1997–1998 school year. This school was selected because it was a distinguished school in the state and noted in the district for its higher mathematics test scores, relative to other schools in the district. (High scores in this district meant performing at the thirtieth percentile.)

In 1997–1998 the school converted to a year-round calendar which resulted in four cross-grade heterogeneous "tracks," each consisting of 13 teachers. To accommodate enrollment, at any one time, three tracks attended school while one was on vacation. The student body, roughly 1300 students, was primarily Latino (90% Latino, 7% African American, 3% Asian American). The transiency rate was approximately 30%. Over 90% of the student body received free or reduced cost lunch. Each classroom was bilingual, but students were transitioned to mainly English instruction in the upper grades.

The group met about once a month throughout the school year after school. Only seven meetings were used for data analysis because the first meeting was an introductory meeting, and teachers did not bring student work. School administrators and support teachers were also invited to the meetings. The principal helped support the occurrence of these meetings by giving up one faculty meeting per month. Thus teachers did not have an extra meeting to attend. The principal and resource teacher attended three meetings. The principal joined each meeting briefly either at the beginning or the end, and she spoke regularly with the research staff.

Workgroups

Student work from teachers' classrooms guided the substance and direction of discussions at each workgroup meeting. Prior to the meetings, teachers used a common problem that they could adapt for their students in their class. For each meeting, teachers selected pieces of student work to share with the group. During the first year, our research team (the authors and two university colleagues) chose these problems ahead of time. The order in which we posed the problems was not set before we started but unfolded based on our reflections on what was happening in the workgroups.[3] The mathematical domains we chose to focus on during the workgroup reflected those that the teachers were working on in their classrooms, such as place value, addition and subtraction, multiplication, and division. Problem types were given to teachers prior to the workgroups and are shown in Table I.

Classroom Visits

To provide ongoing support to the teachers, build relationships, and collect data, we visited the teachers in their classrooms as a means of learning more about student thinking and teachers' practices. Typically, we visited once and frequently twice, between each workgroup meeting. The visits

TABLE I

Student Problems Presented in Order of Workgroup Meetings

Problem type	Problem
W1. Join Change Unknown (JCU)	Ashley has 9 (46)[a] stickers. How many more stickers does she need to collect so she will have 17 (111) stickers altogether?
W2. Measurement Division	1. There are 31 children in a class. If 4 children can sit at a table, how many tables would we need? 2. There are 231 children taking a computer class after school. If 20 students can work in each classroom, how many classrooms would we need?
W3. Multiplication	1. Mrs. North bought 13 pieces of candy. Each piece of candy cost 4 pennies. How many pennies did she spend on candy altogether? 2. Mrs. North bought 15 boxes of animal crackers. Each box cost 47 cents. How much money did Mrs. North spend altogether?
W4. Computation	9 + 7 = ______ 20 + 17 = ______ 28 + 34 = ______ 29 + 16
W5. Compare	Rosalba has 17 (101) bugs in her collection. Hector has 8 (62) bugs in his collection. How many more bugs does Rosalba have than Hector?
W6. Choice of missing addend (JCU), measurement division, or subtraction	1. Yvette collects baseball cards. She has 8 (67) Dodger cards in her collection. How many more Dodger cards does Yvette need to collect so that she will have 15 (105) Dodger baseball cards altogether? 2. Yvette has 34 (274) baseball cards. She wants to put 10 baseball cards in each card envelope (or box). How many envelopes will she need to put away all of her cards? 3. Yvette had 34 (274) baseball cards. She sold 11 (89) of them. How many does she have left?
W7. Final Meeting	Overview of all problems and strategies; no new problems posed.

Note. [a] The numbers in parentheses indicate the larger number size we provided.

TABLE II

Study Participants

Name	Grade	Teaching Experience
Rosalba	K	8
Jazmin	K	1
Miguel	1	0
Paula	1	3
Adriana	1	3
Rose	2	3
Patrick	2	0
Kathy	3/4	0
Anna	3/4/5	0
Natalie	3/4/5	9

were not structured formal observations, but, rather, informal visits.[4] Due to the focus of this particular article, we will not provide direct analysis of our classroom data. However, the role of classroom visits will be apparent in the way the facilitator made use of her knowledge of teachers' classrooms in shaping conversations.

PROCEDURE

The research described in this article involves cross-grade workgroup meetings run by the second author[5] with 10 teachers from one of the tracks in the school. The teachers represented a range of grade levels and teaching experience (see Table II). During the workgroup meetings we encouraged teachers to adapt the problem by changing the number size and context if they felt the changes would be appropriate for their students. We asked teachers, however, to keep the structure of the problem the same. At the beginning of each meeting, teachers briefly reflected in writing about the pieces of student work they had selected to share with the group. They also indicated what problem they actually posed to their class and why they made any changes.

The facilitator then invited teachers to share the variety of strategies that they observed in their classrooms. Teachers could comment on how they adapted the problem, how their students reacted to the problem, and specific ways in which their students attempted to solve it. As the strategies were described, the facilitator recorded them on chart paper so the group could revisit them later in the meeting. The facilitator consistently pressed teachers to describe the details of the students' strategies. The facilitator also introduced common strategies into the discussion if it appeared that teachers had not seen them in their own classrooms.

In order to explore what the strategies revealed about student thinking, the facilitator then asked teachers to compare the relative mathematical sophistication that the strategies demonstrated. For example, a strategy that involves counting by ones from nine to 17 is less sophisticated mathematically than one involving a derived fact, for example 8+8 is 16, 8 is one less than 9 so it would be 17. As teachers voiced their ideas about the strategies, we introduced terminology from the CGI frameworks such as direct modeling and derived facts to label the strategies into working frameworks that revealed the development of students' thinking (see Table III). Occasionally, the facilitator brought in her knowledge of research on children's thinking by elaborating on observations that the teachers made. The group discussed the mathematical principles that underlie the various strategies and what they revealed about students' mathematical understandings. The facilitator redirected questions about particular strategies and their place within the framework to the group for discussion or encouraged teachers to investigate them when they returned to their classrooms. The working frameworks served as a source for continued deliberation, reflection, and elaboration in subsequent meetings as teachers continued to pose problems to their class and learn about their students' thinking.

ANALYSIS

Data Sources

In the larger study, data collection occurred across two settings: the workgroups and classrooms. We documented all the interactions we had with teachers in the workgroups, in their classrooms and in informal interactions. The data analyzed for this article consist of: (a) seven workgroup meeting transcripts from audio recordings; (b) written teacher reflections; (c) copies of student work shared by the teachers; and (d) end-of-the-year teacher interviews.

Data Analysis

The data were collected during a single school year and were managed and analyzed systematically. We drew on case study and grounded theory approaches (Merriam, 1998; Strauss & Corbin, 1988). In the analytic process, we made initial conjectures while analyzing existing data and then continually revisited and revised those hypotheses in subsequent analyses. The resulting claims and assertions are thus empirically grounded and can be justified by tracing the various phases of the analysis.

We, the authors, transcribed the audiotapes from each meeting, collating the written comments made by teachers at each meeting, creating logs and noting major themes. During several initial readings of the transcripts and summaries, we asked questions of the data that centered on building an understanding of how teachers were talking about student work and what kinds of mathematical and pedagogical issues were raised. We created two broad categories that reflected issues raised in the workgroup: (a) understanding student thinking and mathematics, and (b) examining relations between students' mathematical thinking and classroom practice.

We then identified a number of more descriptive themes that consistently emerged and re-emerged across the year, creating focused codes for each of these themes that reflected the content of the conversations. We used the focused codes to code all the transcripts (see Table IV for focused codes). The codes were not mutually exclusive and were applied to exchanges or segments of conversation. An exchange was defined as a unit of conversation centered around the same issue. Thus, if the facilitator asked a teacher to describe a strategy and then there were several turns in which the strategy was detailed, the entire exchange was coded rather than individual turns. Some exchanges had multiple codes. We created a table (see Table IV) following each of our codes across the year. We wrote memos that kept track of the way the focused codes revealed the learning trajectory of the group in relation to the larger themes that were of interest to this study. This was because we were interested in understanding the way teachers' talked about student work and whether there were changes or developments in their interactions across the year. We used the analytic commentaries to articulate how discussions about mathematics, student thinking and pedagogy evolved over the course of the year. We identified the trajectory of two major shifts: attention to children's thinking and developing instructional trajectories in mathematics. Finally, we selected exchanges that were illustrative of the the development of the two major shifts.

FINDINGS

Two major shifts in teachers' workgroup participation emerged from our analyses. The first shift in teachers' participation centered around attending to the details of children's thinking. This shift was related to teachers'

TABLE III

Problem Types and Strategies

Problem	Direct Modeling	Counting	Derived Facts
Join Change Unknown Ashley has 9 stickers. How many more stickers does she need to collect so she will have 17 altogether?	Makes a set of 9 counters. Makes a second set of counters, counting "9, 10, 11, 12, 13, 14, 15, 16, 17," until there is a total of 17 counters. Counts 8 counters in second set.	Counts "9 [pause], 10, 11, 12, 13, 14, 15, 16, 17," extending a finger with each count. Counts the 8 extended fingers. "It's 9."	"9 + 9 is 18 and 1 less is 17. So it's 8."
Separate Result Unknown There were 24 children playing soccer. 7 children got tired and went home. How many children were still playing soccer?	Makes a set of 24 counters and removes 7 of them. Then counts the remaining counters.	Counts back "23, 22, 21, 20, 19, 18, 17. It's 17." Uses fingers to keep track of the numbers of steps in the counting sequence.	"24 take away 4 is 20, and take away 3 more is 17."
Measurement Division There are 31 children in a class. If 4 children can sit at a table, how many tables would we need?	Makes a set of 31 counters. Measures out four counters at a time until all the counters have been used. Counts 7 piles of 4 counters and 1 piles of 3 counters. "We need 8 tables."	Skip counts by 4s until 32, "4, 8, 12, 16, 20, 24, 28, 32" Uses fingers to keep track of the groups of four. "We need 8 tables."	"4 times 6 is 24. 7 more to get to 31. So that's 2 more groups of 4. That's 8 tables. But one table only has 3 kids."
Multiplication Mrs. North bought 13 pieces of candy. Each piece of candy cost 4 pennies. How many pennies did she spend on candy altogether?	Makes 13 piles of 4 counters. Counts them all up by ones. "It's 52 cents."	Skip counts by 4s, 13 times, "4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52. It's 52 cents."	"10 times 4 is 40. 3 times 4 is 12. 40 + 12 is 52."

TABLE III

Continued

Problem	Direct Modeling	Counting	Derived Facts
Compare Rosalba has 17 bugs in her collection. Hector has 8 bugs in his collection. How many more bugs does Rosalba have than Hector?	Makes a row of 17 counters and a row of 8 counters next to it. Counts the 9 counters in the row of 17 that are not matched with the set of 8.	There is no counting analog of the matching strategy.	"8 + 2 is 10 and 7 more is 17. It's 9."

Note. Adapted from: Carpenter, Fennema, E., Franke, M.L., Levi, L., & Empson, S.B. (1999). *Children's mathematics: Cognitively Guided Instruction.* Portsmouth, NH: Heinemann.

attempts to elicit their students' thinking and to their subsequent surprise and delight in noticing sophisticated reasoning in their students' work. The second shift in teachers' participation consisted of developing possible instructional trajectories in mathematics that emerged because of the group's attention to the details of student thinking. The particular mathematical focus was related to the understanding of place value evident in students' ability to decompose and recompose numbers efficiently. In the course of presenting our findings, we also highlight the mediating role of the facilitator and student work in the learning of the group.

Shifting Participation Towards Attention to Children's Thinking

The first major shift in teachers' participation occurred in how they attended to the details of students' mathematical thinking. The way teachers engaged around student work shifted early in our work together; teachers found ways to interact with students about their strategies and to document those interactions for the purpose of sharing in the workgroup. Teachers came to the first workgroup meeting uncertain and unaware of the different ways their students solved the workgroup problems (see Table IV). As teachers continued to try a variety of problems, the focus of the group shifted again towards giving details about what the teachers percieved as more complex student-generated algorithms. Using examples from the data, we show how the substance of workgroup exchanges shift as teachers' engagement with the student work shifts from one of uncertainty about students' thinking to one of active engagement with

TABLE IV

Distribution of Kinds of Exchanges in Workgroup Discussions

Exchange Topic	Workgroup 1 missing addend	Workgroup 2 division	Workgroup 3 multiplication	Workgroup 4 computation	Workgroup 5 compare	Workgroup 6 choice	Workgroup 7 final	Total
Attention to Children's Thinking								100
Detailing strategies	4	12	9	11	8	6	2	52
Noting cumbersome strategies	1	3						4
Students not successful with problem	4	2	1	1	1		9	
Making sense of ambiguous solution	5		2			1		8
How to elicit student thinking	1				1			1
Ts excited by what kids can do		1	5	7	2	2		17
T shares sophisticated strategy	1		1	3	3	1		9
Sharing Success in Practice								5
Anna shares success						1		1
Jazmin shares what she has learned					1	1		2
Patrick shares success						1		1
Paula notes first advanced strategy				1				1
Developing Mathematical Agenda								54
Challenging standard algorithm			2	2	1			5
Discussion of place value			3	5		1		9
F presses issue of place value	1	1	2	6		1	1	12
F clarifies/elaborates CGI framework	3	5	3	2	5	5	5	28

TABLE IV

Continued

Exchange Topic	Workgroup 1 missing addend	Workgroup 2 division	Workgroup 3 multiplication	Workgroup 4 computation	Workgroup 5 compare	Workgroup 6 choice	Workgroup 7 final	Total
Generating Questions about Practice								42
How to run mental math session					1			1
Ts discuss where to go next			1		1	2	1	5
Ts poses question about practice			1	6	8	2	3	20
Ts share ideas for classroom		4	2	1	8	1		16
Other Facilitator Support								46
F shares a student's strategy	1	3	2	1		1		8
F encourages experimentation	1	2	5	4		1		13
F encourages Ts to share		1		1				2
F models interaction with students	1	2	2				4	9
F shares observation from classroom	1	3	7	1			2	14

Note. F = facilitator (second author); T = teacher; Ts = teachers.

student strategies and a recognition of the mathematical competencies the strategies revealed.

Initial participation: Being unaware of the details of students' strategies
For the first workgroup meeting, teachers had posed a join change unknown or a missing addend word problem (e.g., 7 + ___ = 11) to their students. The workgroup meeting began with the facilitator asking the teachers how their students solved the problem. During the course of this meeting, teachers examined their students' work, trying to determine how their students had solved the problem. The first five teachers, who shared their students' strategies, pointed to the incorrect strategies their students used and were surprised that the problem was difficult for their students. Although three out of 16 strategies shared were correct ones, the teacher presenting the work were unsure as to how the students had completed the problem. The teachers then tried to interpret what the student had done based on what was written on the paper. For example, one of Miguel's students had simply written "1 2 3 4 5" on his paper. The group generated several possibilities about how the student had solved the problem. The group concluded that the student had counted out loud "7, 8, 9, 10, 11" but wrote "1 2 3 4 5" to keep track of his count. Miguel, however, could not verify that strategy because he had not seen how the student solved the problem or heard him talk about it.

Generating strategies for eliciting student thinking
In the first two workgroups, teachers worried that eliciting student thinking was a difficult practice. Many teachers relied on their students' written work and did not see it as important to engage their students in conversation about their strategies. Teachers interpreted the direction to pose a problem to their students in ways that inhibited their ability to talk to the students by giving it as a test, sending it home for homework, asking a substitute to give the problem, or giving it as independent work and not circulating to talk to the students. Many teachers worried out loud that they had missed their students' thinking while others thought that the difficulty stemmed from students' inability to articulate their thinking.

Given the general uncertainty about students' thinking, the facilitator made a purposeful move in the first two meetings to engage the teachers in brief discussions about how to elicit student thinking. In fact, as Table IV indicates, much of the facilitator support occurs in the first half of the meetings, modeling and providing support for teachers' interactions in the classroom. The facilitator recognized that eliciting students' ideas would require a change in the ways teachers talked with their students. As a group, the teachers began to notice that the student work did not speak for itself. It could provide a trace of student thinking, but interactions with students were important. In order to raise this issue for everyone in the group to consider, the facilitator drew on her knowledge of what a few teachers were already doing in their classrooms and provided new ideas about questions that might help children explain their thinking.

In the first workgroup meeting [W1], the facilitator explained that students often make very general comments when they are not used to explaining their thinking. Those comments do not reflect a lack of strategy but a lack of experience. Some teachers agreed. Sara added to what the facilitator said by explaining the resistance she experienced when she began to ask children how to explain their thinking.

Sara: Or sometimes too I found that some of my students were, I guess just like, they felt intimidated. They had their right answer, but then you ask them, "how did you get it?" It was like, "I got it, why do you want to know?" They didn't want to say how they got it. (W1: 10/28/97)

Sara's comment made public the idea that students would have to learn that explaining their thinking was both valued and a necessary part of doing mathematics. To help teachers start conversations with their students, the facilitator suggested asking questions such as, "What numbers were you thinking of in your head?" "What number did you start with?" Kathy then shared a strategy she used by explaining that sometimes students can show what they did, even if they cannot describe it verbally very well. For example, students can demonstrate how they counted. For many teachers in the group, eliciting student thinking was a novel practice.

Detailing strategies

The impact of the facilitator's press for details was evident in subsequent workgroup meetings. Some teachers began to draw on annotations they made on the student work to help them remember what their students had said. The following exchange demonstrates Miguel offering a student strategy with more specificity than he did in the first workgroup. The problem involves figuring out how many tables of four are needed for 16 children (See Table I). He refers to the notes he made.

Miguel: So afterwards, one kid really impressed me. John, after he did it – he showed me the answer, and I wrote down what he said. He laid out one crayon and he put four crayons around it. And he represented one table with four students. So he put another crayon out and set the second table. And he put four students there, and the third crayon and he represented four students. Until he got up to 16. He counted up to 16 with the crayons. (W2: 12/9/97)

Detailing more sophisticated student strategies

When teachers posed the multiplication and computation problems in their classes, a subtle shift occurred in the kinds of strategies teachers focused on detailing. The teachers began to recognize that some strategies were quite intricate and amazing because these strategies were not consistently emerging from all classrooms. In the January meeting, four teachers (Kathy, Jazmin, Miguel, and Natalie) brought work that evidenced more sophisticated reasoning. For the multiplication problem, what is the total cost of 15 boxes of animal crackers at 47 cents each?, (see Table I) teachers shared a variety of students' strategies. In Kathy's class, some students added 15 sevens first and then the same number of 40s. Natalie saw similar strategies in her class and she described the ways that students kept track of the 40s and sevens.

Natalie: This one [he counted the sevens] by threes, and this one by twos. He went 14, 14, 14, 14, 14, 14. Since there were only three left because it was an uneven number, the last three he made 21. And then over here with the fours, did 8 and 8. And then put the eights together to make 16. So he's got these rafters going out.

Patrick: It's really wild.

Facilitator: We actually see this strategy a lot. When we let kids invent their own ways – this is one of the most common ways that they do it. Naturally, without us prompting at all, they use this kind of arrow notation. (W3: 1/20/98)

Repeatedly in transcripts, especially in meetings 3 through 5 (see Table IV), teachers expressed their amazement as these strategies were shared, "That's wild!" "How neat!" "Wow!", suggesting that they were surprised and intrigued by students' invented algorithms. In the fourth workgroup, Kathy shared strategies for 28 + 34 and 20 + 17. For 28 + 34, one of her students, Ricardo, had added eight and four to make 12. Then he added the 12 to 30 to make 42 and then 20 more to make 62. For 20 + 17, he added 7 to 20 to make 27. And then he added the remaining ten to make 37. The teachers were amazed but now able to follow both the strategies fairly easily. However, they needed some help with the third one Kathy described.

Kathy: The last one, it started to make sense to me why he was doing this because the last one is 29 plus 16. He switched it to 28 plus 17. Which makes it even to add up to 40. He could add two out of the 17 to make 30. And then add 15. Do you know what I mean?

Teachers: Wait, what! Say that again. I don't understand.
Kathy: You can break the seven apart into a five and a two.
Facilitator: Oh, and he took the two from here and made this a –
Kathy: 30. And add 15 to this to make 45.[6]
Teachers: Ohhhhhh!
Paula: That went over my head!
Patrick: That's amazing!
[A little chatter about that strategy.]
Patrick: Like he had to reason – but he had to reason – see, what's so weird about that is he had to – he's got the 15 and the two on the bottom group there which are easy to add. You make 30 and 45. I mean, but he had to think that the 28 could become 29 and the 17. I'm sorry, the other way around.
Kathy: He can borrow it.
Patrick: But he broke up the seven into five and two. Now how did he think though, I need a seven. So I'll make 16, 17 and 29, 28. That's like, he's going way ahead of his thinking. That's – I mean the rest of it makes sense when you see it like this, but he had to think all that way. He'd be a great chess player.
[Teachers laugh] (W4: 3/3/98)

Paula's comment, "that went over my head," and Patrick's choice of noting "what's so weird about the strategy" suggested that the child's reasoning was atypical and required teachers to slow down and follow it closely (many of them saying in unison, "wait, wait! Say that again"). The sharing of the invented algorithms created both amazement and levity – the teachers were poking fun at their own need to go over the strategies slowly. This further supports the claim that these strategies were not familiar to them. It is important to note that the group was not dismissing the strategies, at least not publicly. Patrick's comment that Ricardo would be a great "chess player" – because he is planning ahead – reflected Patrick's positive regard for the child's thinking.

Observing sophisticated reasoning in primary grades

The shift in noting children's sophisticated reasoning is an important marker of teacher learning because the comments teachers offered in the discussions showed they were impressed with their own students' thinking.

If students in their own classes were using these strategies, then they all should be able to encourage such thinking. This shift in the type of observations made in the later workgroup meetings contrasted with what teachers noted in the first meeting. In that first meeting, in five separate exchanges, teachers focused on how children were unsuccessful with the problem or were using cumbersome strategies. It is also significant to the trajectory of the group that these observations of sophisticated reasoning were not just limited to students in the upper grades. The kindergarten and first grade teachers also observed new competencies in their students. In the second meeting, Miguel declared twice how stunned he was that his first graders were able to solve a division problem and felt he had underestimated them. And Jazmin explained below during the third meeting:

Facilitator: Some of the kids can count by twos.
Miguel: In kindergarten – wow that's great.
Facilitator: Tell them what happened today when you did the two crayons for two cents each.
Jazmin: And then what if I bought three crayons. And they said six. Six cents.
[Teachers impressed, "Wow!"]
Jazmin: Because we thought they would need a lot more visuals and manipulatives. And some of them were like, we don't need it.
Facilitator: One student today made up a problem. Five pieces of gum. Five cents each.
Jazmin: Yeah. Eduardo. And then he had to go back because he put his final answer as five. And I said, "Well how many pieces of gum do you have?" And he said, "Five." And I said, "How much do they all cost?" He said, "Five cents." Well, you have to go back and indicate that because he couldn't count by fives. He went back and he put the five markers for each one. And then he went back and counted them. Each time he put his markers, he would go back and recount them to make sure he had five. Long process. But he got it right. (W4: 1/20/98)

The teachers began to re-evaluate their contentions that particular types of problems belonged to particular grade levels. It is important to note that when Jazmin came to the first workgroup meeting, she was the only teacher who did not bring student work. She expressed then how she was afraid to pose the first problem to her students, thinking that they would be unable to solve the problem.

Making progress: Seeking help and sharing successes of elicitation

The claim that the group was learning to attend to children's reasoning is further supported by the struggles and successes that teachers made public in the workgroup. We see consistent emphasis on detailing across the workgroup meetings – the coding reveals between 4 and 12 exchanges in each meeting to which teachers brought student work (see Table IV). In exchanges in each workgroup meeting, teachers return to voicing both their puzzles and experiments with helping children articulate their thinking. But, eliciting student thinking was not a straightforward task for everyone in the group. Teachers shared two different kinds of observations in the group. The first related to some people's practice of modeling or explicitly showing strategies. The second was working on eliciting strategies in the first place. Next, we provide examples of those.

Noting the impact of teacher modeling on children's strategies

The structure of the workgroup meetings, as we mentioned earlier, remained similar throughout the year. And the bulk of each meeting was spent on describing as many different strategies that students were using. During the fourth meeting, two teachers, Miguel and Paula, made an observation or raised a concern about how their own decisions to model strategies first affected the range of strategies their students subsequently used. We cite these two observations as examples that it mattered to the group that teachers were attending to children's thinking. Two teachers, who believed it was their job to show children strategies first, began to question whether that was necessary.

Miguel: I'm kind of concerned now because I've been teaching my kids one method of adding large numbers. Use numbers like 11 plus 6. I say, take the 11 put it in your head. 6 in your hand. 12, 13, 14, 15, 16, 17, 18. What worries me now from what you're saying, I feel like maybe I'm stunting their ability to group. (W4: 3/3/98)

Miguel's reference to children's ability to group is related to the ongoing conversation the group had been having about children's understanding and use of place value to solve computational problems (an issue we develop further below).

Paula: I find that when I say – if I'm explaining these things to my kids, if I don't say anything at all, and just put it [the problem] up there and say, okay, this is what you need to do. Solve it and be able to tell me how you got the answer, they do much better than if I try to kind of prep them on what they're doing.

[Other teachers nod to indicate they have noticed the same thing.] (W4: 3/3/98)

In the fifth and sixth meeting, there were six exchanges among the teachers about the various sophisticated strategies that children across the grade levels were using. We argue that it was these exchanges that made public the fact that some teachers were noting a greater range of strategies in their classrooms than others. In the fifth workgroup, Anna shared her continued frustrations with eliciting student thinking.

Anna: And then I just have another question. I always feel like when I come in here, I have my stuff and I haven't had time to look at it. So what I'm wondering is any suggestions on like – because I'm trying to get them to write out what they do, so I don't need to sit and talk to everybody. And it's not totally working yet, and I haven't made time to talk to people. So I come in and feel like, you know what, I have all this stuff and I still don't really know what they did. So anyone have any suggestions? (W5: 3/24/98).

In contrast to the beginning of the year, Anna received a barrage of help from the teachers, and notably not just from the facilitator. A number of teachers suggested different ways she could select a few students to observe more closely each day, either by selecting them to share their strategies at the board or asking them questions independently. They gave her suggestions about how to select which students to talk to and what kinds of questions to ask. They also provided several ways of structuring the class period so that students could work on different tasks, while enabling Anna to work with a small group of students.

The sharing of strategies introduced the idea that students had strategies of their own, distinct from teachers' attempts to teach strategies. Being able to detail students thinking implicitly meant that teachers' participation with their students in the classroom was changing. In the penultimate meeting, teachers who had struggled to elicit strategies came back to share successes:

Patrick: A couple of my kids did a subtraction algorithm. Just 15 minus 9. 15 minus 8 equals 7. And those were the only kids who got it right. I wanted them to elaborate on how they did it. How did you know to subtract? I'm just really beginning to get them to explain, even to understand strategies and to explain them. So it's going to be awhile before they're able to articulate it. But I'm encouraged because they're starting to understand that they can articulate how they knew something. (W6: 5/5/98)

....

Anna: . . . I was always having trouble sitting, making time to sit down with them and really listen to what they're saying. Yesterday, I finally did it. Harvey said that he went 5, 10, 15. And then he had 3 fingers, 3 boxes. So of course, I'm like, it's a strategy! And I talked to him! And I listened! And we're writing it down! And I was so excited . . .. (W6: 5/5/98)

Patrick and Anna's contributions to the discussion exemplify that paying attention to the details of children's thinking did emerge as a normative aspect of what it means to contribute to the workgroup. At the beginning of the year, there was doubt in the group that students could explain their thinking. At first, prompted by the facilitator and then supported by several teachers' experimentation in their classroom, some teachers began to share ideas about eliciting students' thinking. By the end of the year, the teachers were sharing the kinds of conversations they had with students that uncovered their thinking and the tasks they used to enable children to express their reasoning. In meetings 4, 5, and 6, as Table IV shows, there were marked exchanges where particular teachers made the kinds of declarations that Patrick and Anna made.

In sum, the first major shift in participation that emerged from our analyses was a shift towards attending to children's thinking. The content of the exchanges shifted towards attention to the details of students' strategies. Initially, the facilitator played a key role in pressing teachers to note the details of children's strategies. This probing also created a need for teachers to elicit children's thinking in their class. Because the content of exchanges shifted, the discourse of the group began to shape a particular stance about the role of teachers, namely, that (a) teachers' work involves attending to children's thinking; (b) teachers make public their efforts to elicit student thinking; and (c) teachers recognize students' mathematical competencies.

Shifting Participation Towards Developing Instructional Trajectories in Mathematics

Our analysis of the data revealed that teachers did not only learn to attend to the details of students' strategies, but also learned that the practice of detailing children's strategies provided opportunities to recognize that students had powerful mathematical ideas. This recognition supported a shift in the group's practices to discuss possible instructional trajectories. We present our analysis of the data to show how the group's discussions led to the identification of particular mathematical ideas for this trajectory. It was how teachers interacted with each other around the student work, again supported by the facilitator, that mathematical goals came into sharper focus for the group. We will show how teachers made use of the details of children's mathematical thinking as they began reconsidering what they wanted to accomplish as teachers in mathematics. Workgroup discussions involved: (a) attending to children's knowledge of place value through a focus on the tens structure of the number system, (b) how to build on students' mathematical thinking, and (c) how to relate students' mathematical understanding to classroom tasks. These discussions contributed to teacher discourse that increasingly centered on instructional trajectories in mathematics. As we examined the workgroup participation in relation to those ideas we saw that while the conversations evolved, the evolution was not linear. As indicated in Table IV, the issues do not build, then peak at one point in time, become resolved and completely disappear. Rather, ideas about instructional trajectories enter into conversations at different points during the year and they come up repeatedly. We found that the instructional trajectories developed in relation to other aspects of the teachers' experiences in the workgroup and in the classroom.

Attending to children's knowledge of place value through a focus on the tens structure of number system

The facilitator's moves, early in the workgroup meetings, helped to surface a mathematical direction for the workgroup conversations. As we described earlier, Table IV shows five separate exchanges in the first meeting during which teachers shared students' unsuccessful or cumbersome attempts at the problem. Two of the teachers, however, identified a strategy that took advantage of the tens structure of our number system. Kathy explained that two of her students "estimated" to find the difference between 48 and 111. They added 60 to 48 to get 108 and then added three more to get to 111. More commonly she saw her students use tallies to count up from 48 to 111 without organizing them into rows of ten. Kathy noted that students who used that strategy often miscounted. A few of Rose's students used base ten blocks to create a set of 48 by putting out 4 ten blocks and 8 units. Yet she too noticed that some went on to count by ones while only a few actually used the tens blocks to solve the problem. The students who used tens to count up from 48 to 111 were generally more successful.

The facilitator capitalized on the sharing of those strategies to introduce the idea of using tens and ones to solve the problems. She asked the teachers to generate direct modeling strategies for solving problems by ones and by tens. After some of strategies were generated, she then suggested ways teachers could support students to progress from direct modeling by ones to direct modeling by tens.

While the idea of using tens in direct modeling was introduced early, the teachers had not begun to explore the use of tens in students' invented algorithms. This makes sense in light of the fact that, with the exception of one strategy, teachers did not observe any invented strategies. The teachers initially thought that the invented strategies were linked to students' "exposure" to them or to how "smart" students were. For many members of the workgroup, the invented strategies were not typical of the way they themselves, let alone their students, would have solved the problem.

Recognizing students' use of ten to solve problems also meant connecting that idea to teachers' conventional views on place value. During the second meeting, Miguel expressed his fear of teaching place value, a topic he had heard from other teachers was notoriously difficult. The facilitator responded by characterizing the strategies that were shared during that meeting as evidence of place value understanding, pushing the idea that place value was not about identifying the hundreds' place. She explained,

> But a lot of what the kids are doing with this problem is place value. Figuring out how many 20s are in 231 is place value. You can think about division as putting things in groups. Place value is putting thing in groups, but it's putting things into groups of ten. . . . I have 89 pieces of candy, and I want to give ten to each teacher, how many teachers can I give them to? . . . It gives them a context in which they have to figure out how do I take 89 and break it down into tens and ones (W2: 12/9/97).

Miguel interpreted place value instruction and understanding as children's ability to identify hundreds, tens, and ones column. The facilitator used Miguel's concern to open up the idea that children's work with groups of tens, through the problem contexts that teachers were posing to their students, was already laying a foundation for understanding place value.

Beginning to think about how to build on students' thinking

Some exchanges in the workgroups involved hypothetical discussions about how to help students move on in their ideas. In the first three meetings, in four separate exchanges, the facilitator encouraged teachers to consider place value understanding as they thought about the next steps with students. For example, in the second workgroup, the problem was to find how many classrooms were needed to allow 231 children to take computer classes, if only 20 children could be in a room. The facilitator asked teachers how they would support students who were direct modeling using ones. She directed the teachers to think about using an element of the student's strategy instead of imposing or asking students to do something that was not connected to their initial strategy. The facilitator provided ideas for teachers to take up and try in their classroom. She provided suggestions for questions that teachers could use when they interacted with students and helped teachers consider how to help students advance their strategies. Thus, while the group was learning to pay attention to the details of their thinking, the facilitator was already prompting teachers to think about how they might respond to help students advance their strategies.

Relating students' mathematical understanding to classroom tasks

The discussions about building students' place value understanding in the classroom peaked in the fourth and fifth workgroups. There were 14 exchanges combined across both workgroups in which a teacher posed a question or an issue about practice to the group.

The fourth workgroup was a watershed – the teachers posed straightforward computation problems, and Kathy came to the group with a host of sophisticated strategies. The facilitator, in a surprised tone, asked her, "how come you're getting all of these strategies all of a sudden?" Kathy laughed and said "they (the kids) went on vacation!" But she went on to describe some of her general uses of problem solving while Patrick asked her questions about the materials she made available to students and what she emphasized. Drawing on a number of linked exchanges, the excerpts below show how Kathy responded:

Kathy: I've just consistently done word problems every day. Plus mental math every day. And encouraging them to solve it two ways. Show their work. . . . They have options to use tens and ones [base ten blocks]. And they used those at the beginning of the year to help them count. . . . They have the option to use that or they can draw a picture, whatever they wanted. And so they got a lot of work with that. And then a lot of them would still go to the algorithm. And I guess whenever they brought it to me, I would say, "what is this?" . . . I mean a couple of times in front of the class, I've said people have shared their strategies. . . . And I'll say, "What did you really add?" So there's just a lot of reinforcement of, "What are these numbers? Is this really a two or is this a 20?" (W4: 3/3/98)

The tone of the discussions shifted again because teachers began to be more interested in how to create opportunities for students to generate their own efficient strategies. They shared their own attempts and looked to members of the groups that were more successful eliciting students' strategies. Mental math activities had been adopted by a number of teachers, and they used their experiences with that task in their classroom as they responded to questions. In the next exchange during the fourth workgroup, Patrick wanted his students to start using a "bar" to represent ten, rather than ten individual marks. Kathy responded by suggesting particular ways he might be able to move students towards that benchmark. The facilitator's role in this conversation was quite limited and provides evidence that the teachers were developing and sharing their expertise in the workgroup. It also provides evidence that detailing students' strategies was being coupled with developing a mathematical instructional trajectory.

Patrick: It would be nice if somebody would just make a one bar. I'm waiting for the kid that makes a ten bar, that draws a line and says that's a ten bar. And then makes the three for the 13. And I've been trying to reinforce that, and they're just not doing that. And I even write these visually. I write numbers on the board with bars and dots. And I say, what is that? And they'll say 43. And they've got it. And I'll do 26 and 43. How much is it? It's 69. And they can do that or whatever the answer was. But then they won't do it themselves.

Kathy: Do you give them numbers that are even ten, like adding and subtracting? Because I was at the same point as you. Like, I don't know when I was doing that. I was doing a lot with the bars.

Patrick: Like 30 minus ten.

Kathy: Yeah. And basically using numbers that were even ten and then larger numbers. And so they sort of –

Patrick: Can you give me an example?

Facilitator: 120 minus 60.

Kathy: Yeah, something like that. Or even stay under 100. Like 80 minus 20. Where they see that using the tens might be easier. Just so they get used to using them. I think I did a lot of that, and then they used the tens more. But some of them still go back to the ones. I mean, even some of them where you can circle it. Can you group these instead of making slash marks? And they don't use the strategy (W4: 3/3/98).

In the exchange, Kathy appeared confident in giving suggestions to Patrick. Yet she also had questions about helping students reach benchmarks which she shared with the group later on in this meeting. She noted that some students in her class still make tally marks when they encounter large numbers. Later in the meeting, she asked for suggestions about helping them make their strategies with larger numbers more efficient.

During the next meeting, the group returned to mental math activities and discussed ways they implemented mental math in their classrooms. Patrick, again, prompts the group to explain how the task is structured in the classroom.

Patrick:	How do you do the mental math thing because I've seen Karla (another teacher not in this workgroup but in the school) do it? How do you do it?
Adriana:	I do it exactly like she does. I got the idea from her! [Laughs]
Facilitator:	This idea is spreading.
Kathy:	My kids love it! We have professor of the day.
Patrick:	Can you just model it for me? You have a group of kids and you say, "Who wants to do a problem?" (W5: 3/24/98)

For the next 18 turns, Adriana and Kathy explained the logistics of how to organize a mental math activity as Patrick asked them more questions about how to do this. This talk also produced new ideas about using student thinking. For example, towards the end of the exchange, the facilitator raised the question of whether the students have an opportunity to ask each other questions, knowing this is a strategy that supports dialogue in the classroom. Later in the meeting, Patrick asked what the goal was of having students solve problems. Kathy responded, "To show their thinking . . . I care about what you're doing in your head." Other teachers contributed to Kathy's response, and the discussion then moved to how teachers structured the time when students shared strategies in front of the class. Three teachers described their various classroom management strategies while Patrick and Anna asked questions.

These technical questions about tasks show that teachers were experimenting in the classroom, and moreover, because several teachers had begun to use similar structures, they could compare the impact on student thinking. It was not just a matter of sharing the latest technique because the teachers wanted to help their students begin to develop efficient computational algorithms. The shift in detailing strategies and noting students' sophisticated strategies also motivates a shift in making practice more transparent. The classroom has to be represented and cannot be directly viewed in the workgroup meeting. Patrick's question about mental math marks the need to explain practice; his question emerges as a question about 'tell' me how you're doing mental math because 'mental math' itself is not transparent (see also Little, 2002).

Summary of Shifts over Time

We have been concerned thus far with the shifts evident in the nature of teachers' participation over time. By using exchanges from the work-group conversations, we have shown how the group's attention shifted with respect to children's thinking, the mathematical ideas at work in children's computational strategies, and the respective questions about practice that those observations generated. Figure 1 summarizes the trajectory of the workgroup over the course of the year.

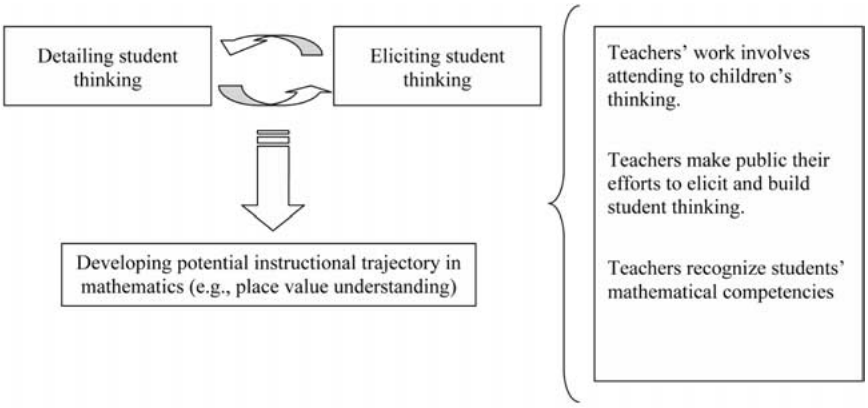

Figure 1. Summary of shifts in participation across Year 1.

DISCUSSION

This paper is an effort to document teacher learning through shifts in participation in regular workgroup meetings focused on examining student work. The workgroup was a setting where teachers shared student thinking and made public their classroom practices. By struggling to make sense of and to detail their students' thinking, the teachers' participation developed the intellectual practices of the workgroup. We focus our discussion on the significance of the use of student work and a transformation of participation view in analyzing the learning trajectory of the group.

Student Work as a Tool for Learning

A tool or artifact can provide a means through which participants in a community of practice negotiate meaning (Wenger, 1998). The work-group conversations revolved around student work, an artifact of students' mathematical thinking, which then opened a window into each teacher's classroom. The common problems that teachers posed to their students allowed teachers to focus on shared meaning, build common ground and negotiate crossing the boundaries of the workgroup meetings and their classrooms. The artifacts supported the development of a shared language that, in turn, contributed to the construction of workgroup meeting practices.

We agree with Ball and Cohen (1999) that "simply looking at students' work would not ensure that improved ways of looking at and interpreting such work will ensue," (p. 16). Because the use of student work is being advocated in current conversations about professional development, it is important to underscore the role that student work played in our work. All the student work came from teachers' own classrooms and thus each teacher could speak to how the work was generated and had opportunities to return to their classrooms to clarify their understanding of student thinking or to extend it. They all had instructional practices that they made explicit and on which that they could build. During the workgroup meetings, the facilitator and the teachers used what was present or not in student work to initiate discussions of student thinking, mathematics, and pedagogy. Centering the activity on teachers' own student work allowed for conversations that deepened as well as challenged teachers' notions about their work as teachers. They developed more detailed knowledge of their *own* students' mathematical thinking and began to articulate benchmarks in the learning trajectories for their students and instructional trajectories to support their work. The student work also allowed the teachers' to begin to see themselves as mathematical thinkers when they were willing to struggle through student strategies they did not understand.

Learning as Participation

Understanding learning as changing participation is significant to our analysis in this paper. We tracked changes in teacher learning by examining shifts in the practices of the workgroup. The teachers' experiences with their students and their shared experiences with their colleagues influenced the form and direction of the workgroup meetings. "When individuals participate in shared endeavors, not only does individual development occur, but the process transforms (develops) the practices of the community" (Rogoff, Baker-Sennett, Lacasa, & Goldsmith, 1995, pp. 45–46). Although we have not attempted here to describe individual teacher change across the workgroups, it is evident from our analyses that certain teachers were actively contributing their methods of experimentation to the discussions (e.g., Kathy) while others struggled more to elicit their students' thinking in the first place (e.g., Anna). It is important that some teachers experimented successfully with the workgroup problems in their classrooms while others struggled. The questions, confusions, and successes teachers shared made certain ideas public that helped shape the focus and trajectory of the group. In this article, we have made the trajectory of the group the main focus of our analysis.

We wish to consider how a transformation of participation perspective strengthens our understanding of teacher learning. Clearly, we can assess changes in teacher knowledge by relying on pre/post measures of individual teachers' knowledge and beliefs. We do not argue that examining individual teachers developing knowledge and beliefs is unimportant. In fact, these are key resources for a developing community (e.g., Even & Tirosh, 2002; Leinhardt & Smith, 1985). However, we believe that by attending to shifts in participation, we can understand the following aspects of teacher learning: (a) how teachers working together supported the development of each other's thinking and the practices they used in their classrooms; (b) how and when teachers asked each other for help and contributed to discussions in the workgroup because of their own experimentation in the classroom; and (c) how teachers looked at the strategies students in other classrooms used and then used those as markers for what to expect from their students.

These aspects of the workgroup practice contributed to the development of a particular kind of intellectual and professional community for this group of teachers. Their shared experience was beginning to develop ideas about instructional trajectories for developing student math concepts, at least with respect to students' fluency with place value. That is not to say that they agreed with one another or had reached a consensus about future directions or their roles within the classroom and the school. The knowledge and beliefs that teachers constructed, however, emerged from their contributions to the creation and continual development of the practice of workgroup meetings and their classroom communities. It is important to understand how teachers participate in developing practice in order to know how to help support teachers' engagement with student thinking, mathematics, and pedagogy. Paying attention to the kinds of shifts that may take place as teachers first begin to work together can also help us identity key markers of generative professional learning structures within schools.

Central to a transformation of participation perspective is that shifts in participation are in service of new roles and identities. Our analysis of the first year of workgroup data leads us to conjecture about the new kinds of identities teachers may be forming through the ways classroom teaching was portrayed in the discussions (Little, 2002). First, teachers were experiencing new ways of working together around a particular focus towards a long-term goal – building children's understanding of the tens structure of our number system in order to develop their fluency with computation and their understanding of operations. Second, the group was exploring new ways of being – teachers elicit and listen to children's mathematical ideas, interpret them, and use resources to decide where to go next to develop ideas. Third, the teachers were finding ways to experiment within their own classrooms and use the workgroup as a place to further reflect on their experimentation.

We have only seen the beginning of the teachers' development of these practices. As the teachers worked together to begin to create a community of learners around the teaching and learning of mathematics, they were also beginning to create a set of norms about what it meant to teach at their school. Continued longitudinal work will help us understand the extent and significance of teachers' changing roles and identities.

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NOTES

[1] Japanese Lesson Study is a form of professional development practiced most commonly among elementary school teachers in Japan. It is a process through which teachers analyze and develop classroom lessons together.

[2] Join Change Unknown is a term used by CGI researchers to categorize addition and subtraction problem types. The word, "join," refers to a joining or combining action in the word problem. "Change unknown," refers to the location of the unknown in the

word problem, in this case an unknown addend. See Carpenter et al. (1999) for the full categorization scheme.

[3] We began with a join change unknown (JCU) or a missing addend problem to introduce teachers to children's mathematical thinking. Many adults see the problem as a subtraction problem, but children will often use an adding or joining strategy to find the missing addend (Carpenter et al., 1999). For that reason, researchers refer to addition and subtraction problems by indicating whether there is a joining or separating action. Most teachers across the grade levels were working on addition and subtraction in the first trimester, and we wanted them to consider how the two operations are related since many textbooks separate the study of addition from subtraction. We moved next to multiplication and division contexts. To many teachers, especially in the primary grades, we suspected that the division and multiplication contexts would appear too difficult since their students would not have started instruction in those areas. However, we wanted them to have opportunities to see that all of the children from kindergarten through grade five would be able to solve multiplication and division problems. We would then be able to talk about direct modeling strategies (in which students represent each number and model the action in the problem) and how they cut across mathematical operations.

We returned to the addition and subtraction context but through computation problems as the teachers approached standardized testing time. We wanted to have conversations about how students could continue to use the same strategies that they used in word problem contexts for computational problems. We also wanted teachers to have an opportunity to see how writing problems vertically versus horizontally would affect the kinds of strategies students used. Finally, we hoped teachers would notice how that students can use a variety of strategies to solve computational problems. We then moved on to compare problems to help teachers think about how action in the wording of the problem may make a problem more or less difficult for students to solve. In the final two meetings of the year, we revisited earlier problems to have some closure about the principles we had learned throughout the year.

[4] Our informal conversations with teachers during these visits were consistently focused on their students' thinking and the evolving frameworks that we constructed in the workgroups. However, we did model how to elicit student thinking through our informal interactions with their students and the questions we asked students during those interactions. On occasion, we suggested additional problems teachers might want to try with their students. We also shared interesting strategies that we encountered as we talked with their students.

[5] The research reported in the article is part of a larger study of teacher learning and change. There were four workgroup meetings taking place in the school, each led by a different member of the research team. The authors of this article led workgroup meetings centered on student work. We chose to focus our findings and discussions around one workgroup using student work for pragmatic reasons.

[6] An easier strategy may have been to just take one from 16 and add to 29 to make 30. Then add 30 and 15. Kathy, however, reported that this student first changed the numbers to 28 and 17.

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